VIBROGRAPH USED AS A VISCOMETER

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Plates XII and XIII.

ABSTRACT. A method has been described for comparing the viscosities of highly viscous liquids by using an improved form of the vibrograph devised by the author. It is essentially a damping coefficient determination method, the damping coefficients K_0 and K being calculated from the vibration curves of an oscillating sphere in air and in the liquid respectively. The method has the merit that it takes a very short time (a fraction of a second), to make an observation and that a permanent photographic record of the damped oscillations is obtained by which the visual uncertainty in noting the ends of the swing is automatically eliminated. It is suggested as an easy and quick method for comparing the viscosities of highly viscous liquids. An experiment conducted with glycerine at various temperatures shows a linear dependence of ηd (viscosity $\times$ density of the liquid) on the value of $(K-K_0)^2$.

INTRODUCTION

The coefficient of viscosity of liquids has been determined by different methods by different investigators. The capillary flow method due to Poiseuille is a very convenient and accurate method. But in the case of highly viscous liquids like glycerine, it takes hours to make even one set of observations of the flow, since the rate of flow is extremely small under ordinary experimental conditions. The method is therefore tedious and unsuitable for the determination of η, the coefficient of viscosity of liquids in such cases. A modified method of using Poiseuille's apparatus for the determination of η of highly viscous liquids is described by Bhimasenachar.[1] The essential features of the arrangement are provision for temperature control, provision for keeping the apparatus dry in the case of hygroscopic liquids and provision and the use of a reservoir into which air is pumped to a required pressure. It takes comparatively smaller time to obtain a reading with the arrangement. Another satisfactory method for the determination of η of highly vicous liquids is the coaxial cylinder method. Among the logarithmic decrement methods for measuring the η of liquids we may mention the well known Meyer's oscillating disc method which consists in finding the logarithmic decrement of an oscillating flat disc in air and in the liquid; the method of simple pendulum vibrating in the liquid (Stokes[2]); and H. Martin's[3] experimental method based on Stokes' mathematical investigation of a vibrating wire in a liquid medium. The accuracy of the method depends on the accuracy of the determination of the damping coefficient of the vibration of an electrically maintained wire in the liquid under test. The chief source of error in the method of finding η based on the determination of κ the logarithmic decrement or the damping coefficient is consequent on the uncertainty in noting the ends of the swings of the vibrating body. In the present investigation, a method has been described which is specially suited for the experimental comparison of η of highly viscous liquids like glycerine. It is essentially a damping coefficient determination method, the damping coefficient being calculated from the vibration curves of an oscillating system in air and in a liquid surrounding the body. The vibration curves are obtained by employing the improved vibrograph designed by the author.[4] The merits of the method are: (1) it takes a very short time, a fraction of a second to make an observation by this method; (2) a permanent photographic record of the damped oscillations is obtained and the visual uncertainty in noting the ends of the swing is automatically eliminated. It is suggested as an easy and quick method for comparing the viscosities of highly viscous liquids.

EXPERIMENTAL ARRANGEMENTS AND THEORETICAL CONSIDERATIONS

The experimental arrangement consists of a steel bar of uniform cross-section rigidly clamped at one end so that a convenient length of the bar projects from the top of the table. A hollow brass sphere about one inch in diameter is rigidly attached to the free end of the bar through a short, stout wire, thereby constraining the sphere to vibrate only in the vertical plane in a direction perpendicular

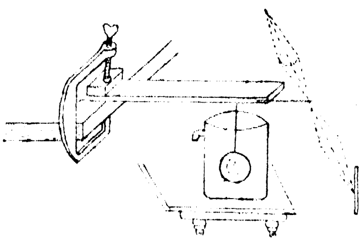


FIG. 1

to the length of the bar. A rigid pointer, which can conveniently be a pin, attached to the free end of the bar as shown in the diagram of the experimental arrangement illuminated by the optical system and focussed on the slit of the vibrograph (*vide* Plates XII & XIII). The bar is set vibrating and the vibrations of the oscillating system are recorded on the photographic plate shot at right angles to the direction of vibrations. Vibration curves are obtained first with the sphere oscillating in air and then in the liquid contained in a vessel placed beneath the sphere so that it is well immersed in the liquid. In working with different liquids the level of the liquid in the vessel is kept the same so that the vibration curves are obtained under identical conditions, as far as possible, the only variables being η and d the density of the liquid when working with different liquids. From the vibration curves, κ_0 the damping coefficient in air and κ the damping coefficient when the sphere is vibrating in the liquid are determined. Starting from the undisturbed position of the vibrating system, if a_0 is the amplitude of vibration at the end of a swing and a_n the amplitude at the end of n oscillations, then $a_0/a_n = e^{\kappa\tau n}$ where τ is the period of oscillation of the system.

$$\kappa = \text{the damping coefficient} = \frac{1}{n\tau} \log_e \frac{a_0}{a_n}.$$

or

$$\kappa = \frac{2.303}{n\tau} \log_{10} \frac{a_0}{a_n}.$$

$$\therefore \qquad \kappa \propto \frac{1}{n} \log_{10} \frac{a_0}{a_n}. \qquad \dots \quad (1)$$

The reaction of the surrounding medium, *viz.*, the liquid in which the sphere is immersed, has two-fold effects. The first is the loss of the energy due to the viscosity of the medium and the second the diminution of energy due to the added mass, a layer of the medium moving bodily with the sphere. This added mass increases the inertia of the moving sphere without increasing its weight. Thus the added mass of the liquid affects the damping constant and not the frequency of the vibrating sphere. The irregularities attending the initial motion of the sphere inside the liquid, the eddies and the wall effect etc., do not affect the periodic time but may produce only second order effects on account of the shortness of time (about 1/5 sec.) during which the observations are recorded. The effect of the wire to which the sphere is attached on the liquid is likewise a small quantity and has a negligible effect on the motion of the sphere. The damping coefficient κ is, as in the case of Martin,[3] a function of η, d, a, the radius of the sphere, d_0 the density of the material of the sphere and N the frequency of oscillations of the vibrating system.

$$\kappa = \phi\,(\eta, d, d_0, a, N).$$

Since, in using the apparatus, vibration curves are obtained with the sphere oscillating in different liquids at the same temperature or in a given liquid at different temperatures, d_0, a and N can be considered to remain constant. We are therefore interested in the investigation of the dependence of κ on η and d. From Martin's experimental verification of Stokes' formula, we are justified in assuming that $\kappa^2 \propto \eta d$ or $\kappa^2 = A\eta d$ where A is a constant. A is not dimensionless but is a function of d_0, a and n.

Thus to compare the viscosities η_1 and η_2 of two liquids having densities d_1 and d_2, we have to determine κ_1 and κ_2 the damping coefficients in the two liquids. If κ_0 is the damping coefficient in air, we have then,

$$\frac{\eta_1 d_1}{\eta_2 d_2} = \frac{(\kappa_1 - \kappa_0)^2}{(\kappa_2 - \kappa_0)^2}. \quad \ldots \quad (2)$$

The effective damping of the liquid only is taken as in Meyer's[5] experiment, the damping due to mechanical causes and vibrations of the sphere in air being subtracted.

EXPERIMENTAL RESULTS

Bi-distilled glycerine for analysis (sp. gr. 1.23), is the liquid chosen and the vibration curves are obtained with the sphere oscillating in the liquid at various temperatures. Table I gives the analysis of the results obtained.

TABLE I

T°C Temp. of the liquid.	a_0 in cm Amplitude at the commencement.	a_n in cm. Amplitude at the end of n (=5) oscils.	Damping Constant κ of the Oscil. system in the liquid.	κ_0 Damping constant in air.	$(K-K_0)$	$(K-K_0)^2$
In air at 32°C	0.93	0.70	...	0.02468	...	...
34.0	0.75	0.35	0.06620	0.02468	0.04152	0.0017240
45.0	1 10	0.59	0.05410	0.02468	0.02942	0.0008654
55.0	0.93	0.52	0.04850	0.02468	0.02382	0.0005665
63.5	0.97	0.58	0.04468	0 02468	0.02000	0.0004000
79.0	0.85	0.54	0.03940	0.02468	0 01472	0.0002167
95.0	1.00	0.66	0.03610	0.02468	0.01232	0.0001518

Note.—In evaluating K_0 and R the constant terms (2,303 and τ) are omitted. This will not affect the final results since we are *comparing* the damping coefficients.

The vibration curves obtained at different temperatures are shown in the plate at the end of the paper. To verify the results indicated by formula (2), the relative viscosities of the same specimen of the liquid at various temperatures are determined by using a viscometer.

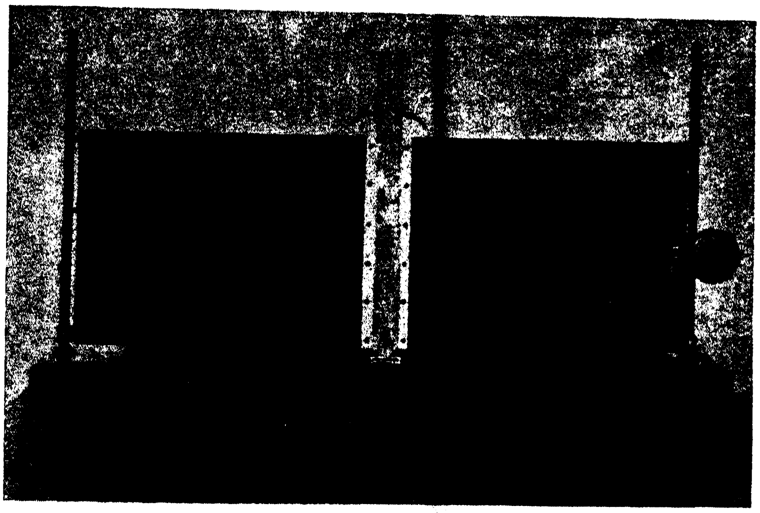


Fig. II. Front view of the vibrograph.

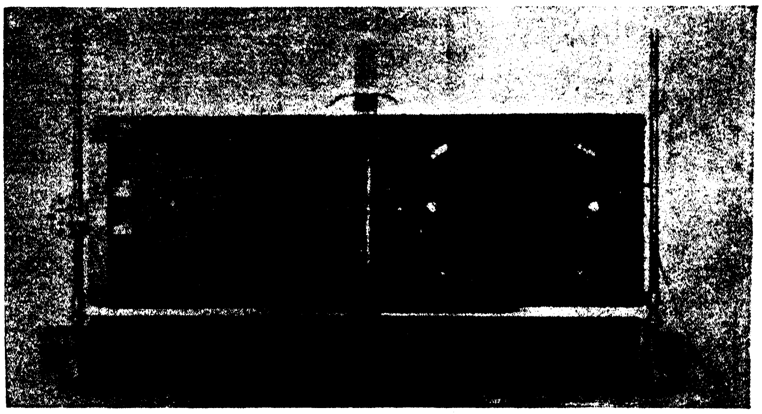


Fig. III. Back view of the vibrograph.

TABLE II.

T°C Temp. of the bath in which the viscometer is immersed.	t secs. Time taken for the liquid to flow past the two marks on the viscometer	d Density of the liquid at the Temp.	$\eta \propto td$ Relative viscosity of the liquid at the Temp.	$\eta d \propto td^2$.
32.0	4854	1.209	5869	7096
40.2	2971	1.203	3575	4300
45.5	2211	1.200	2653	3184
58.0	1271	1.192	1515	1805
69.0	890	1.185	1052	1946
80.0	612	1.177	720	847
95.0	397	1.166	463	540

Table II gives the values of relative viscosities obtained. A graph (Fig. 2) is drawn showing the relation between temperature T and ηd, and from the gralhp, the values of ηd corresponding to the temperatures at which $(\kappa - k_0)^2$ are obtained by the vibrograph are computed.

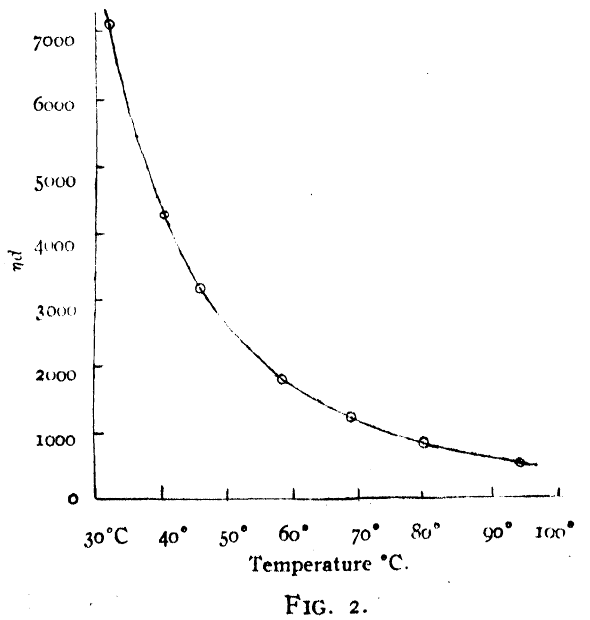


FIG. 2.

Table III gives the interpolated results.

TABLE III

T°C						
Temperature of liquid ...	34·0	45·0	55·0	63·5	79·0	95·0
Relative values of ηd at T°C got by interpolation from Fig. IV. ...	6250	3250	2050	1500	880	540
$(K-K_0)^2$ from vibration curves. ...	0.0017240	0.0008654	0.0005665	0.0004000	0.0002167	0.0001518

Fig. 3 shows the linear relation between ηd and $(\kappa-\kappa_0)^2$ both computed at the same temperature.

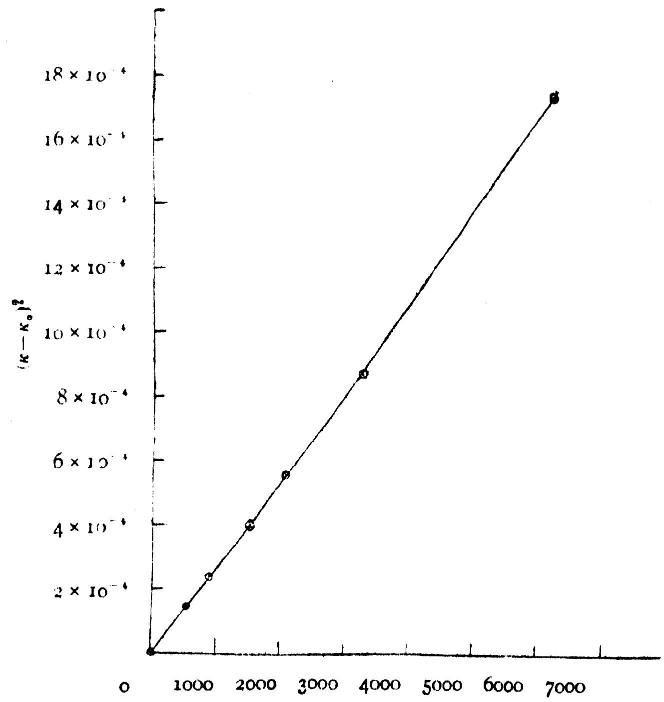


FIG. 3

The experimental results obtained with the liquid therefore support the procedure suggested for comparing the viscosities of liquids using a vibrograph. This method of comparing the viscosities of liquids using the vibrograph where for

Air 32°c

Ko = ·02468

(K—Ko)

liquid 95°c ·01232

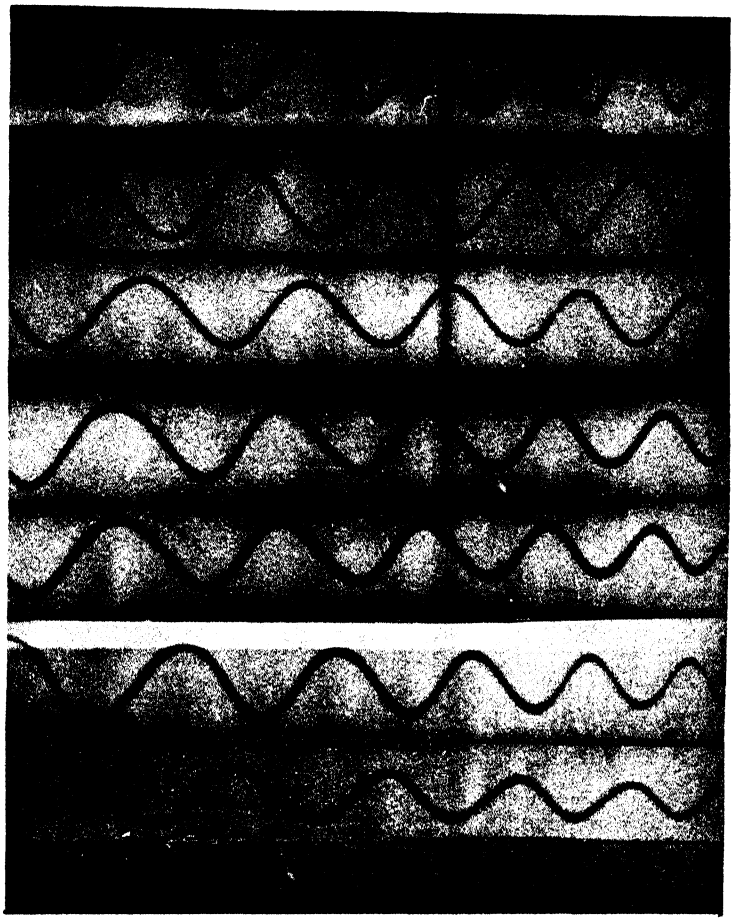
79°c ·01472

63·5°c ·02000

55°c ·02380

45°c ·02942

34°c ·04152



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each liquid the time required is of the order of less than a second will be found convenient in the quick determination of relative viscosities of thick oils etc., used for lubrication purposes; especially so because of the permanent photographic records of the vibrations which can be used for subsequent computation or verification.

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